Technical Note No. 105

STRAIN GAGE OUTPUT IN RESPONSE TO FREE THERMAL EXPANSION, NON-THERMAL STRESS, AND THERMAL STRESS

Introduction

It is often the goal of the stress analyst to determine the state of stress in a part or structure so that a comparison to some stress-based failure criterion (e.g. yield stress, maximum principal stress, von Mises, etc.) can be conducted. For an isotropic material over its elastic range, the state of stress can be obtained using experimental strain data and the biaxial[1] form of Hooke's Law.

Since a lot of experimental strain data is obtained using the electrical resistance strain gage it is important to understand the output of the strain gage in response to 1) free thermal expansion, 2) non-thermal stress, and 3) thermal stress.

Strain Gage Thermal Output

The output of the electrical resistance strain gage is a function of the change of resistance,ΔR, of the grid conductor. Contributions to ΔR can be separated into two classifications:

I. Mechanical (i.e. stress-induced) strain, $\varepsilon = \Delta L/L$, of the substrate (i.e. the material to which the gage is bonded).
II. Temperature change of grid conductor and substrate.

The latter of these two classifications is manifested as *thermal output*[2] and is quantified by the following equation[3]:

$$\left[\frac{\Delta R}{R_0}\right]_{TO} = \left[\beta_{SG} + F_{SG}\left(\frac{1+K_t}{1-\nu_0 K_t}\right)\left(\alpha_{SUB} - \alpha_{SG}\right)\right]\Delta T$$

[1] Most likely a *biaxial* (not uniaxial) state of stress exists on the surface of a test part or structure. Thus the acquisition of proper strain gage data requires the use of a two-element TEE rosette aligned with the principal stress axes. In case the directions of principal axes are unknown, a three-element rosette is required. With rosette data in hand, the *biaxial* form of generalized Hooke's Law can then be used to calculate the principal stresses - see the Appendix on page 8..

[2] In the past, resistance changes due solely to temperature change were referred to as apparent strain, but by international agreement the proper term to use now is thermal output.

[3]From Measurements Group Tech Note TN-504 S*train Gage Thermal Output and Gage Factor Variation with Temperature*. This equation assumes free thermal expansion (contraction) and therefore contains a correction factor that accounts for the transverse sensitivity of the strain gage subjected to equally biaxial substrate expansion (contraction) - more about this on page 4.

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where:

ΔR is the change of resistance due to change of temperature.

R_0 the initial gage resistance and the subscript *TO* signifies Thermal Output.

β_{SG} is the temperature coefficient of resistance of the strain gage grid conductor.

F_{SG} is the strain gage's Gage Factor as provided by the strain gage manufacturer.

Kt is the transverse sensitivity of the strain gage.

ν_0 is the Poisson ratio of the standard test material used by the manufacturer in calibrating the gage to determine the Gage Factor.

$(\alpha_{SUB}-\alpha_{SG})$ is the difference in thermal coefficient of expansion between the substrate and the grid respectively.

ΔT is the temperature change from some initial reference.

Note there are two fundamental contributors to thermal output,

a) The grid resistance change due to the temperature dependent resistivity of the grid conductor.

b) The relative difference in thermal coefficient of expansion between the grid and the substrate.

Item a) is self-explanatory; item b) deserves a little more of our attention:

To the degree that the coefficient of thermal expansion of the grid and substrate differ, *there will be a mechanical strain induced in the grid.* Since the grid is by design sensitive to mechanical strain, a corresponding resistance change will occur and thus contribute to thermal output.

Free Thermal Expansion

Ideally, the strain gage output that results from free thermal expansion (also understood to include contraction) should be zero. The reason this is desirable is because there is no stress associated with free thermal expansion. Furthermore, with no output due to free thermal expansion, there will be no "contamination" of any gage output that *does* result from stress whether the stress is non-thermally induced, thermally induced, or a combination of both.

For the case of free thermal expansion, *both a dimensional change and a temperature change occur coincidentally*. Because no output is desired in this case, the gage manufacturers have developed special grid alloys that result in near zero output under the condition of free thermal expansion. In this case, as the grid and substrate it is bonded to change in dimension *and* temperature, ΔR as expressed in the thermal output equation given on page 1 remains nearly zero. These special alloys are employed in what are referred to as self-temperature compensated (STC) gages. It follows that in order to achieve the STC functionality, the gage alloy needs to be tailored to the coefficient of thermal expansion of the substrate. For example, in the case of a substrate made of steel, gages with a specified STC of 6 ppm/°F should be selected because this most closely matches the coefficient of thermal expansion of steel. Strain gage manufacturers also typically provide a data sheet with STC gages that shows thermal output versus temperature[4]

[4]Improved accuracy can be obtained if test part temperature is available. In this case thermal output data provided by the gage manufacturer can be subtracted from the test data. Another technique is to temperature soak the test part while keeping it mechanically and thermally stress free and thereby experimentally determine thermal output as a function of temperature and then use this data for subsequent correction. Yet a third technique is to configure a bridge system that includes a thermally integrated *and* stress free dummy completion gage.

Non -Thermal Stress

Non-thermal stress as defined here concerns a dimensional change of material due to *application of a mechanical stress at constant temperature*. We just saw above that for an STC gage installation undergoing free thermal expansion (or contraction), a near zero output is produced when a change of both substrate dimension and temperature occurs. Therefore, it follows that a non-zero output will result with the same change in substrate dimension (albeit this time via applied stress) and no change in temperature. The gage output that results in this case is related to the mechanical (stress-induced) strain in the substrate, $\varepsilon = \Delta L/L$, and this strain is related to the applied stress via Hooke's Law.

Thermal Stress

This case concerns a third permutation of dimension and temperature change (or lack thereof): specifically, *no allowed change of dimension (i.e. a constrained substrate) with an accompanying change of temperature*. Assuming a temperature compensated gage installation and, using similar reasoning as above, we expect (and get) a non-zero ΔR output. What does this gage output represent? A simple thought experiment gives the answer: the output is the same as one would get if the part were allowed to freely expand due to an identical temperature change, and then, while held at that final temperature, squeezed back to the exact dimension of the original constraints by applying a mechanical compressive stress. The output from the last step in this thought experiment is of course the compressive strain related via Hooke's Law to the applied mechanical compressive stress. To help convince one's self that this is indeed the answer consider this: *The strain gage neither knows or cares how the part it is bonded to arrives to a thermally-changed and physically-constrained state of existence,* be it path 1: constrained, then heated; or path 2: free, heated, then mechanically stressed back to the initially constrained dimension of path 1. Figure 1 on page 7 should help to clarify the definitions and relationships of free thermal expansion, non-thermal stress, and thermal stress.

Before we leave the case of thermal stress as manifested by a constrained substrate that undergoes a temperature change, it is instructive to delve a little deeper into the specific source of the gage's output. Recall that the output of a strain gage can be separated into three sources: 1) mechanical (stress-induced) strain of substrate, 2) temperature-induced resistance change of the grid conductor, and 3) difference of coefficient of thermal expansion between grid and substrate - the latter two, as we have seen, being contributors to *thermal output.* Let us now perform a direct comparison of free thermal expansion versus uni-axial thermal stress (uni-axially constrained part) for each of these three sources:

- Free Thermal Expansion

1. Mechanical (stress-induced) strain in substrate = 0 (there is a dimensional change associated with *thermal strain* but since stress = 0 there is no associated stress-induced ΔL dimensional change, therefore mechanical strain $\varepsilon = \Delta L/L = 0$)..
2. Grid resistance change = $[\beta_{SG}]\,\Delta T$
3. Difference of coeff. of thermal expansion between substrate and grid = $\left[F_{SG}\left(\frac{1+K_t}{1-\nu_0 K_t}\right)(\alpha_{SUB}-\alpha_{SG})\right]\Delta T$

For an STC gage installation, 2) and 3) are nearly equal and opposite so as to produce the desired near zero output.

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- **Uni-Axial Thermal Stress (Uni-Axially Constrained Part with Temperature Change)**

1. Mechanical (stress-induced) strain in substrate = 0 (although now stress ≠ 0, mechanical strain is still = 0 since part is constrained i.e. ΔL = 0 therefore mechanical strain ε = ΔL/L remains = 0)
2. Grid resistance change = $[\beta_{SG}]\Delta T$ (this value remains the same as for the free thermal expansion case)
3. Difference of coefficient of thermal expansion between substrate and grid.

Now items 2) and 3) are not nearly equal and opposite. This is because the coefficient of thermal expansion of the substrate along the grid axis is now effectively equal to zero due to it being constrained - and therein lies the source of non-zero gage output for the case of thermal stress. To illustrate: as the constrained substrate is heated, the strain gage grid attempts to expand against a substrate that now has an effective coefficient of thermal expansion equal to zero. This action manifests as compressive mechanical strain in the grid and as we have seen from the thought experiment described earlier, the resulting output of the gage is equivalent to the compressive strain related via Hooke's Law to the thermally induced stress in the substrate.

To further illustrate the point:
Using the Thermal Output equation given on page 1 for the case of steel with α_{SUB} = 6 PPM/˚F we have:

$$F_{SG} * \varepsilon_{free} = \frac{\Delta R}{R_0} = \left[\beta_{SG} + F_{SG}\left(\frac{1+K_t}{1-\nu_0 K_t}\right)\left(6_{PPM} - \alpha_{SG}\right)\right]\Delta T$$

This equation contains a correction factor (the quotient within the parenthesis). Correction is needed due to the transverse sensitivity (K_t) of the gage. The correction factor accounts for both: 1) the fact that the thermal strain induced during free thermal expansion is biaxial and equal in both directions and 2) the manner in which the gage was calibrated to obtain F_{SG} (i.e. in a uniaxial stress field on a material with Poisson ratio = ν_0). Since K_t is relatively "very small" the quotient within the parentheses is ≈ 1. Therefore for the purpose of this discussion we will consider transverse sensitivity of the gage to be negligible. Setting K_t = 0 and distributing terms the above equation can be rewritten as:

$$F_{SG} * \varepsilon_{free} = \frac{\Delta R}{R_0} = \beta_{SG}\,\Delta T + F_{SG}\,6_{PPM}\,\Delta T - F_{SG}\,\alpha_{SG}\,\Delta T$$

Dividing through by the gage factor F_{SG} and assuming for the sake of this discussion a“perfectly” self temperature compensated (STC) gage, i.e. one that gives exactly zero gage output for a given ΔT of free thermal expansion,
we get:

$$\varepsilon_{free} = \frac{\beta_{SG}\ \Delta T}{F_{SG}} + 6_{PPM}\ \Delta T - \alpha_{SG}\ \Delta T = 0 \qquad \text{(eq 1)}$$

Let us now express the gage’s output that corresponds to the condition of heating a piece of steel constrained only along the direction of the grid axis of the gage. We will call the gage output in this case ε_{con}. This case is tantamount to an imaginary material such that α_{SUB} = 0 PPM/˚F along the grid axis albeit retaining in the transverse direction an α_{SUB} = 6 PPM/˚F and the “freedom” to move in that direction. We therefore recognize that there will exist a component of substrate displacement (thermal strain) transverse to the gage grid due to the free thermal expansion in that direction in addition to the Poisson strain (also in the transverse direction) created by the thermal stress due to the constraint. But, since we assume K_t = 0, the adaptation of the Thermal Output equation for this case is straightforward. Proceeding then with the Thermal Output equation to express the gage output of our imaginary material
we get:

$$\varepsilon_{con} = \frac{\beta_{SG}\ \Delta T}{F_{SG}} + 0_{PPM}\ \Delta T - \alpha_{SG}\ \Delta T \qquad \text{(eq 2)}$$

Taking the difference between the free thermal expansion gage output (ε_{free}) and the gage output due to heating the constrained piece of steel (ε_{con}) we can eliminate the β_{SG}, F_{SG} , and α_{SG} terms in equations (1) and (2) and since we assumed ε_{free} = 0 we now have:

$$\varepsilon_{con} = -6_{PPM}\ \Delta T \ = -6\mu\varepsilon\ \Delta T$$

This is the result we expect from our thought experiment where the free thermally expanded material is "squeezed back" along the direction of the grid axis to its original dimension. In other words, during free thermal expansion the piece "grows" along the grid axis an amount equal to 6 PPM/˚F and thus the required "squeeze-back" along the grid axis needs to be of an equal and opposite amount of microstrain. Based on the thought experiment alone the reader should be convinced that a more rigorous treatment that accounts for $K_t \neq 0$ as well as non-perfect STC behavior* will not detract from this result.

* See footnote on page 2

Summary

In summary, the temperature compensated electrical resistance strain gage:

1. Will produce zero (or near-zero) output in response to free thermal expansion. There are several practical techniques to compensate for non-zero free thermal expansion output. Zero or "near-zero" output in response to free thermal expansion is desirable because there is no stress associated with free thermal expansion.

2. Will respond to non-thermal stress in terms of stress related strain as manifested by the stress-induced ΔL/L dimensional change in the substrate. This strain output is related to the stress via Hooke's Law.

3. Will respond to thermal stress in terms of stress-related indicated strain as manifested by strain gage thermally induced output and *this strain data is related to the thermally induced stress via Hooke's Law.*

In the most general case, the surface of a part or structure will experience some combination of free thermal expansion, non-thermal stress, and thermal stress. The fact that the temperature compensated electrical resistance strain gage effectively ignores free thermal expansion and produces output due to both non-thermal and thermal stress in terms of strain data that can be used to calculate total surface stress via the biaxial form of Hooke's Law is an elegant and useful feature that should not go unappreciated by the stress analyst.

ADVISORY ON THE USE OF EXPERIMENTAL STRAIN DATA TO CORRELATE A FINITE ELEMENT MODEL (FEM) FOR A THERMALLY LOADED STRUCTURE

As we have seen, the temperature compensated strain gage: 1) does not produce output when a structural dimension change occurs due to free thermal expansion and 2) does produce output when no structural dimension change occurs due to thermal stress. In light of this it is advised that correlation of an FEM with experimental strain data for a thermally loaded structure *not* be done on a strain-results basis. The experimental strain data should be transformed into the stress domain and the correlation then performed on a stress-results gage rosettes and the biaxial form Hooke's Law can achieve the transformation of the experimental strain data to stress data.

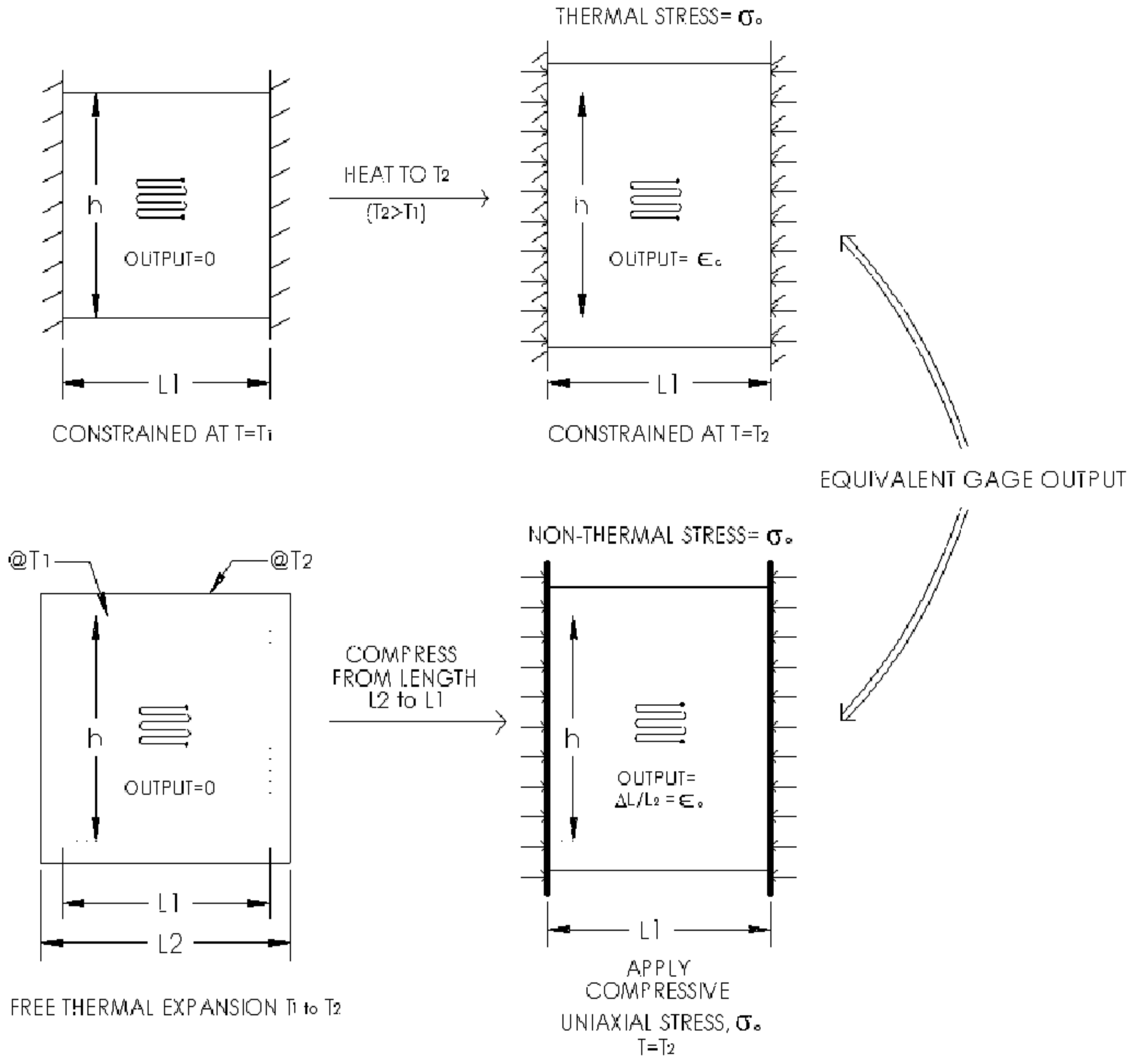

Figure 1. Illustrative relationship between free thermal expansion, non-thermal stress, and thermal stress with regard to STC gage output (where the STC feature here exactly compensates for free thermal expansion).

APPENDIX

The biaxial form of Hooke's Law:

$$\sigma_P = \frac{E}{1-\nu^2}\left(\varepsilon_P + \nu\varepsilon_Q\right)$$

$$\sigma_Q = \frac{E}{1-\nu^2}\left(\varepsilon_Q + \nu\varepsilon_P\right)$$

where:

ε_P and ε_Q are the principal strains ***that can be derived from rosette strain gage data****

E is modulus of elasticity

ν is Poisson's ratio

σ_P and σ_Q are the principal stresses

A few words about the von Mises Failure Criterion:

Of the several stress-based failure criteria, the von Mises criterion agrees the best with failure-by-yielding in ductile materials.

The von Mises criterion accounts for combined stresses and is based on material distortion or change of shape and thus takes into account failure due to shear stresses. This is why it is the best failure criterion to use for ductile materials.

In practice, the von Mises "stress" (a calculated value with units of stress) is compared to a benchmark, namely the ductile material's *tensile yield strength*. A von Mises stress value less than the material's *tensile yield strength* is desireable. A von Mises stress value equal to the material's *tensile yield strength* indicates impending material yield.

As mentioned above, the von Mises criterion takes into account shear stress. To check this, consider the engineering "rule of thumb" that "*shear yield strength* is typically ~ 60% of *tensile yield strength* for ductile materials". This rule of thumb is supported by published shear and tensile yield strengths of ductile materials.

The biaxial form of the von Mises stress is:

$$\sigma_{VM} = \sqrt{\sigma_P^2 - \sigma_P\sigma_Q + \sigma_Q^2} \qquad \text{with } \sigma_P \text{ and } \sigma_Q \text{ principal stresses}$$

Consider the condition of pure-shear. A quick inspection of Mohr's circle for this condition shows that the principal stresses are equal and opposite to one another, and furthermore, equal to the magnitude of pure-shear stress. Let us now normalize the principal stresses with respect to the aforementioned *tensile yield strength* benchmark. We do this by making $\sigma_P = 1$ and $\sigma_Q = -1$. This gives a $\sigma_{VM} = \sqrt{3} = 1.73$ meaning that when the magnitude of the principal stresses equals the *tensile yield strength*, the von Mises stress *exceeds* the *tensile yield strength* benchmark by a factor of 1.73 (not good). This means in order for the von Mises stress to *equal* the *tensile yield strength* benchmark, the magnitude of the principal stresses must be reduced by a factor of 1/1.73 or 58%, and since the magnitude of the principal stresses equals the magnitude of the pure-shear stress, the magnitude of pure-shear stress to cause impending yield is equal to 58% of the *tensile yield strength*. Thus the establishment of a link between the above stated rule of thumb and the von Mises failure criterion.

*See for example Measurements Group Tech Note TN-515 Strain Gage Rosettes-Selection, Application and Data Reduction

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